Abstract Title Page

Title: Mediation Effects of Latent Numerical Abilities on the Associations between Domain General Competencies and Fraction Knowledge

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Abstract Body

Background / Context:

Fraction knowledge is a crucial component of overall mathematical proficiency, and competence with fractions is central to subsequent mathematics skills, such as algebra and geometry (e.g., Booth & Newton, 2012; National Mathematics Advisory Panel [NMAP], 2008). Despite fractions' vital role, however, many students struggle to understand fractions (e.g., Hecht, Close, & Santisi, 2003; NMAP, 2008; Siegler et al., 2012). For example, only 49% of fourth-grade students correctly identified the number of fourths in one (National Assessment of Educational Progress (NAEP), 1996), and 50% of eighth graders failed to correctly order from smallest to largest the fractions 2/7, 5/9, and 1/12 (Martin, Strutchens & Elliott, 2007). Thus, there is a pressing demand to understand both the direct and indirect sources of difficulty and early indicators of fraction understanding.

Explanations of numerical development, for both whole numbers and fractions, have evolved over the years. The first account (Ni & Zhou, 2005; Vosniadou, Vamakoussi, & Skopeliti, 2008) proposes that the acquisition of whole number and fraction knowledge is a segmented process, in which learning about whole number knowledge is acquired first and fractions are learned later, with great difficulty. Ni and Zhou (2005) call this interference the *whole-number bias*. Alternatively, recent accounts (Siegler & Lortie-Forgues, 2014; Siegler, Thompson, & Schneider, 2011) propose an integrated developmental theory that emphasizes a key developmental continuity across all types of real numbers. Under this integrated theory, the real source of fraction difficulties stem from the failure to accurately represent and arithmetically combine the magnitudes of fractions. To better understand how early numerical competencies support the development of fractions knowledge, it is important to examine whether the developmental of these two forms of mathematics skills share commonality on early domain general processes, and whether, whole number skills will interrupt the associations between fraction knowledge and their shared domain general antecedents.

Several components are necessary to understand a complete developmental theory of and individual differences in fraction learning. The conceptual framework in this study rely on the conceptual and procedural mathematics knowledge that rooted in Geary's (2004) model of mathematics learning. In the context of fractions, conceptual knowledge involves an understanding of multiple representations of fractions, their magnitudes, and the relationship between the numerator and denominator; while procedural knowledge pertains to the fluency with the four fraction arithmetic operations, i.e., addition, subtraction, multiplication, and division of fractions and mixed numbers (Siegler et al., 2013). The second component involves the non-symbolic and symbolic magnitude representations (Siegler et al., 2013). Non-symbolic magnitude representations refer to the understanding of concrete stimuli (e.g., which rectangle shows 1/3 shaded?); symbolic magnitude representations pertain to the understanding of a conventional representation (e.g. which number is bigger, 3/4 or 1/2?). As the centrality of knowledge in numerical development, magnitude understandings, including both symbolic (i.e., number line estimation) and non-symbolic magnitude representations (i.e., non-symbolic proportional reasoning) are independently and relatedly important in supporting fraction learning (Siegler & Lortie-Forgues, 2014). A third component involves the compelling evidence that whole number skills (e.g., multiplication and division) are of particular importance to proficiency in fractions. Division is important because fractions and whole number division are logically equivalent (Behr & Post, 1992; Siegler & Pyke, 2013). The ability to see multiplicative relationships may help students when working with equivalent fractions (Boyer & Levine, 2012). A complete developmental theory should also consider cognitive and behavioral characteristics of children that engender individual differences on the acquisition with some kinds of math knowledge (Ackerman, 1999; Hecht, 1998; Hecht et al., 2003). Prior research provide sufficient empirical supports for the influence of domain general cognitive processes on whole number competencies (e.g., Fuchs et al; 2010; Geary et al., 2008; LeFevre et al., 2010), on fraction outcomes (Fuchs et al 2013; Hansen et al., 2015; Hecht & Vagi, 2010), and both (e.g., Bailey et al., 2014; Hecht et al., 2003; Namkung & Fuchs, 2015; Vukovic et al., 2014).

Purpose / Objective / Research Question / Focus of Study:

The proposed study intended to unravel the underlying developmental pathway from domain general cognitive processes and early mathematical skills to the acquisition of fraction knowledge after the bulk of fraction instruction takes place. Despite of an ample quantity of studies examining the domain general source of individual difference in whole-number or fraction skills, no consensus yet has been reached regarding whether fraction learning relies primarily on early domain general predictors, or whether it has early precursors only as an extension of previous mathematical skills on whole numbers. This means that domain general competencies may be important for the development of fraction knowledge only in that they promote the acquisition of intermediate whole number related skills, which in turn, are direct precursors of children's understanding of fractions. The purpose of the present study was to reveal the developmental pathway from third grade cognitive competencies to sixth grade conceptual and procedural fraction knowledge through the intervening whole numerical skills at fifth grade. Fraction outcomes were operationalized as both procedural understanding and computation proficiency. We were particularly careful in choosing the whole number related skills: both symbolic (i.e., whole number line estimation) and non-symbolic magnitude representations (i.e., proportional reasoning) were considered to be independently important; arithmetic proficiencies in whole numbers were operationalized as long division and multiplicative reasoning skills. Four domain general competencies that were suggested to represent important sources of individual differences were: attentive behavior, language ability, non-verbal reasoning, and working memory. In addition, demographic characteristics (i.e., age, gender, and income) were controlled as covariates. Given the integrated theory of numerical development as well as the solid foundation of cognitive precursors for both whole number and fraction knowledge, it is hypothesized that whole number related skills interrupt the associations between the shared cognitive indicators and fraction outcomes. It is also hypothesized that differential pathways for procedural and conceptual knowledge will be identified, that is, some cognitive predictors may influence fraction concepts or fraction procedures through a selected combination of underlying state of whole number competencies.

Population / Participants / Subjects / Data Collection:

This study used empirical data that come from 536 students in nine schools across two Delaware public school districts. Students were followed longitudinally for four years, beginning in third grade and ending in sixth grade. All students in participating schools were taught with curricula aligned with the Common Core State Standards in Mathematics (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). Demographic information (i.e., gender, age, and income status) was assessed at third grade and was provided by the school districts (see Table 1). The four domain general cognitive predictors were measured at third grade using the following instruments: SWAN Rating Scale for attentive behavior (Cronbach's α = .98; Swanson et al., 2006); The Peabody Picture Vocabulary Test (PPTV) for language ability (α > .96; Dunn & Dunn, 2007); Wechsler Abbreviated Scale of Intelligence (WASI) for nonverbal ability (α > .90; Wechsler, 1999), and Working Memory Test Battery for children (WMTB-C) for working memory (Test-retest reliability = .61; Pickering & Gathercole, 2001). The mediator variables were assessed in fifth grade: locating 22 whole numbers on a 0-1000 number line adapted from Siegler and Opfer (2003) for number line estimation (α = .91); 48 randomly ordered trials adapted from Boyer and Levine (2012) for non-symbolic proportional reasoning (α = .93); six long division problems (α = .76) and three multiplicative skills (α = .59). The fraction outcomes were assessed in sixth grade: three shaded fraction items from Hecht et al. (2003) and 25 items from National Assessments of Educational Progress (NAEP, 2009; α = .86) for fraction concepts; 26 fraction computation items adapted from Hecht (1998; α = .82) for fraction procedures.

Significance / Novelty of study:

Studies examining developmental predictors for fraction knowledge either neglected the possible pathways via which domain general characteristics of children may affect individual differences in fraction outcomes (e.g., Bailey et al., 2014; Jordan et al., 2013; Namkung & Fuchs, 2015; Seethaler et al., 2011), or subjected to a limited inclusion of outcome measures and intermediate components (Vukovic et al., 2014), or implemented a cross sectional mediation design that is inadequate to depict a longitudinal sequence of numerical development in which cognitive precursors support fraction learning via intervening numerical skills during fraction instruction (Hecht et al., 2003). We believe that the previous developmental models require further refinement. On the one hand, without considering a wide range of mediators, the direct effects from general domain predictors would be overestimated while the mediation power of whole-number related skills would be underestimated, resulting in a biased pathway that might lead to a false evidence for *whole number bias*. On the other hand, studies that had included multiple highly correlated mediators using observed scores in the mediation models might lead to the “multicollinearity” problem under the context of mediation, that is, even though the model will count for the total variances captured by the entire bundle of mediators, but it blurs the extent to which each mediator contributes to the total variances due to “information redundancy”. The issue of multicollinearity in empirical studies using linear regression has been well diagnosed and remedied by the implementation of methods such as ridge regression, principal component regression, or partial least square regression. However, educational researchers who intended to build the mediation models for the development of fractions did not give the issue enough attention. This methodological flaw might be blamed for previous discrepant findings on the development pathways, i.e., the “intervening role” of whole-number skills and potentially differentiated individual pathways through which domain general competencies influenced fraction skills. The current study is the first attempt to demonstrate the differentiated development models by overcoming these existing methodological flaws.

Statistical, Measurement, or Econometric Model:

Mediation analysis, following a sequential structural equation modeling (SEM) approach, was used to examine the relationships among third grade general cognitive competencies, fifth grade whole number related skills, and sixth grade fraction knowledge. Mediation analysis allows for a simultaneous investigation of the direct associations between third-grade cognitive competencies and sixth-grade fraction knowledge, and, importantly, how these associations are mediated by whole number related skills measured at fifth grade. Simultaneously considering a fuller set of important cognitive indicators, whole-number mediators, and fraction outcomes offers the advantage of providing a more accurate and stringent test of each ability’s contribution because each variable competes for variance against other constructs. To improve the methodological limitations, latent mediation variables were identified representing underlying constructs of whole number related skills, we believe these latent mediation variables will not only offer a more reliable representation of the state of whole number skills at the intermediate stage, but also overcome the multicollinearity issue. Following Anderson and Gerbing's (1988) recommendations, we analyzed the data in two stages: a measurement model analysis stage and a structural model analysis stage. In the measurement model analysis stage, we conducted two sets of confirmatory factor analyses to determine the factor structure among the four whole-number mediators and the two fraction outcomes, respectively. In the structural model analysis stage, we assessed the mediating pathways through the latent mediators obtained from stage one. Independent models would be analyzed for the latent fraction outcome, as well as for the two components of fraction knowledge, respectively. The hypothesized three models are shown in Figure 1.

Usefulness / Applicability of Method:

The aforementioned empirical data were analyzed to achieve the research goal. Statistical analyses were performed by entering the covariance matrix of all variables to the *Mplus* statistical program (Muthén & Muthén, 1998-2014). Before conducting the structural models, we examined the significance of all the direct relationships between the predictors, latent mediators, and outcome variables to make sure they meet the three prerequisite criteria for determining a consistent mediation model as defined by Baron and Kenny (1986). The relative magnitude or unique contributions of a mediator is decided by the corresponding effect size (where .05 = a small effect size, .09 = a medium effect size, and .25 = a large effect size, according to Keith, 2006; Kenny, 2014). In the measurement stage, two correlated dimensions, or factors, were identified for the four whole-number related skills. Long division and multiplicative skills mapped on to the same factor, which we conceptualized as “whole number arithmetic”. Whole number line estimation and non-symbolic proportional reasoning mapped onto a separate factor, which we conceptualized as “magnitude understanding”. The model-data fit of the two-factor model was significantly better than an alternative the one-factor model (see Table 4). Therefore, the two factor scores were incorporated as latent mediators into the structural model analysis stage. The two fraction outcomes (fractions concepts and fractions procedures) loaded on to one dimension, or factor, which can be conceptualized as “fraction knowledge”. The fraction knowledge factor captured 78.4% of the variance in fraction concepts and 43.5% of the variance in fraction procedures, which is thus be considered as a valid measure representing general knowledge of fraction. In addition, it is of interest to examine the two components of fraction knowledge separately in order to satisfy our second purpose of identifying any differentiated pathways from cognitive competencies to fraction knowledge through a selected combination of underlying state of whole-number competencies. Towards this end, although the two fraction outcome measures loaded very well onto one factor, we included three structural models in which the fraction knowledge factor, fraction concepts, and fraction procedures were used as outcome variables, respectively (see Figure 1).

Findings / Results:

Table 2 presents the means and standard deviations of all measures. Table 3 presents correlations among measures used in the study. The three models all demonstrated a good fit (see Table 4). All direct relationships amongst predictors, mediators, and the outcome measure (*a* paths, *b* paths, and *c* paths in Baron & Kenny, 1986) were significant except for those non-verbal reasoning and working memory predicting fraction procedures (see Tables 5 and 6 for the results of prerequisite regression analyses). All direct path coefficients from the predictors to fraction knowledge outcome were insignificant, while all coefficients relating both mediators to both predictor and outcome variables were significant (see Table 7 for path coefficients). This indicates that the relationships between the general cognitive predictors and the fraction knowledge outcome were fully mediated by the combination of the two latent whole-number mediators. In order to compare the relative and unique contributions of the two mediators in the mediation effect, we examine their effect sizes in the model. The mediation effect sizes through whole number arithmetic were small-to-medium for language, nonverbal reasoning, and working memory (ES = .07- .08), while medium-to-large for attentive behavior (ES = .21). The mediation effect sizes through magnitude understanding were medium-to-large for all predictors (ES = .23-.29). The relations between all the cognitive predictors and fraction concepts were fully mediated only by the magnitude understanding (mediation effect sizes were medium-to-large for all cognitive predictors, ES = .25 - .32). The relations between both attentive behavior as well as language ability and fraction procedures were fully mediated only by the whole number arithmetic (magnitude understanding was not a statistically significant predictor of fraction procedures). The mediation effect sizes through whole number arithmetic were medium for language ability (ES = .09) and medium-to-large for attentive behavior (ES = .24).

Conclusions:

By using a sequential SEM design, we found that the effects of all the four domain general predictors on general fraction knowledge were fully mediated by the latent state of whole number magnitude understanding (i.e., the centrality understanding of symbolic and non-symbolic) and that of whole number arithmetic (i.e., the commonality of division and multiplicative reasoning). In another words, domain general competencies is important for developing fraction knowledge only in that they are direct precursors for intervening whole-number related skills, which in turn promote understanding of fractions. More specifically, it is through whole number magnitude understanding, rather than through whole number arithmetic, that cognitive competencies indirectly influenced six graders' knowledge in fraction concepts; attentive behavior and verbal ability at early stages are important for developing whole number arithmetic, which, in turn, are relevant for learning fraction procedures. Our study is the first of its kind to find full mediation effect of whole number skills on the cognitive-fraction association, which resonates with the integrated theory of numerical development, i.e., a unitary process underpinning the development of all types of numerical knowledge. It is also the first disclosure that conceptual and procedural knowledge in fraction have its unique antecedents in whole number skills: magnitude understanding (also a conceptual type of knowledge) and arithmetic (also a procedural type of knowledge), respectively. Such identification also provide important practical guidance. For example, the recognition in the types of important domain general competencies lends evidence to develop screening tools to identify at-risk students for early intervention, the recognition of the important role of whole number learning inform numerical instruction during the intermediate grades.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Table 1 *Sample Demographic Information (N = 536)*

Characteristic	%
Gender	
Male	47.0
Female	53.0
Race	
White	51.9
Black	40.0
Asian/Pacific Island	5.7
American Indian/Alaskan Native	2.5
Hispanic	17.7
Low Income	60.9
English Learner	10.6
Special Education	10.6
Mean Start Age in months	105.9

Table 2 *Means and Standard Deviations of All Measure*

Variable	M (SD)	*n*
Cognitive Predictors		
Attentive behavior (SWAN)	36.76 (21.01)	468
Language ability (PPTV; percentile)	47.16 (28.63)	464
Nonverbal ability (WASI; scaled score [M=10])	9.81 (3.26)	462
Working memory (WMTB-C)	19.38 (21.31)	460
Numerical Skills Mediators		
Whole number line estimation (PAE)	8.43 (5.67)	407
Non-symbolic proportional reasoning	.70 (.21)	401
Long division	3.82 (1.84)	407
Multiplicative skills	2.28 (.92)	401
Fraction Outcome		
Fraction concepts	21.25 (5.55)	361
Fraction procedures	11.70 (5.08)	361

Note: All scores are raw scores unless indicated otherwise.

Table 3 *Correlations among All Measures*

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Age	--												
2. Female	-.032	--											
3. Low income	.140*	-.036	--										
4. Attentive behavior	-.156*	.173*	-.257*	--									
5. Language ability	-.076	-.053	-.385*	.374*	--								
6. Nonverbal ability	-.118*	.077	-.268*	.412*	.480*	--							
7. Working memory	-.151*	-.042	-.098*	.340*	.268*	.352*	--						
8. Whole number line estimation	.248*	.122*	.287*	-.377*	-.433*	-.433*	-.375*	--					
9. Non-symbolic proportional reasoning	-.097	.010	-.214*	.380*	-.346*	.346*	.251*	-.403*	--				
10. Long division	-.207*	-.027	-.189*	.460*	.337*	.374*	.345*	-.450*	.380*	--			
11. Multiplicative skills	-.250*	.061	-.100*	.479*	.264*	.305*	.274*	.376*	.309*	.548*	--		
12. Fraction concepts	-.266*	.053	-.252*	.524*	.519*	.498*	.405*	.647*	.516*	.541*	.504*	--	
13. Fraction procedures	-.146*	.060	-.191*	.474*	.326*	.304*	.270*	.430*	.366*	.511*	.505*	.603*	--

Note. Whole number line estimation is coded as percent absolute error, therefore, higher scores indicate poorer performance. Low income is indexed by participating in free or reduced lunch.

$^*p < .05$.

Table 4 *Model Fits and Model Comparisons for the Measurement Models*

Model	*df*	χ^2	*p*	RMSEA	CFI	TLI	SRMR
Measurement Models							
Two-factor mediators	1	.030	.863	.000	1.000	1.018	.001
One-factor mediator	2	11.377	.003	.107	.971	.913	.030
One-factor outcome	0	.000	.001	.000	1.000	1.000	.000
Structure Models							
Predicting fraction knowledge	28	86.9	.001	.063	.946	.900	.037
Predicting fraction concepts	17	27.4	.052	.034	.986	.963	.022
Predicting fraction procedure	17	27.7	.048	.034	.988	.968	.021

Note: *df* = degrees of freedom; RMSEA = root mean square error of approximation; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardized root mean square residual.

Table 5 *Multiple Linear Regression of Cognitive Predictors on Numerical Competency Mediators and Fraction Outcomes*

	Fraction Concepts		Fraction Procedures		Numerical Reasoning		Whole Number Calculations	
Predictors	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value
Attentive Behavior	.281 (.046)	.001	.364 (.053)	.001	-.258 (.063)	.001	.449 (.057)	.001
Language Ability	.308 (.051)	.001	.157 (.062)	.012	-.264 (.065)	.001	.169 (.061)	.006
Nonverbal Ability	.174 (.050)	.001	.039 (.059)	.516	-.261 (.064)	.001	.142 (.061)	.020
Working Memory	.112 (.045)	.013	.057 (.054)	.289	-.219 (.057)	.001	.155 (.055)	.005
Covariates	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value
Age	-.172 (.040)	.001	-.065 (.047)	.160	.166 (.051)	.001	-.195 (.049)	.001
Gender	.005 (.040)	.893	.005 (.047)	.914	.166 (.051)	.001	-.060 (.049)	.223
Income Status	.030 (.043)	.487	-.007 (.050)	.894	.111 (.054)	.040	.061 (.053)	.246

Note: Estimate is standardized coefficients; S.E. = standard error.

Table 6 *Multiple Linear Regression of Numerical Competency Mediators on Fraction Outcomes*

	Fraction Concepts		Fraction Procedures	
Predictors	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value
Numerical Reasoning	-1.075 (.267)	.001	-.230 (.168)	.173
Whole Number Calculations	-.124 (.246)	.615	.525 (.148)	.001
Covariates	Estimate (S.E.)	*p*-value	Estimate (S.E.)	*p*-value
Age	.002 (.055)	.970	.095 (.047)	.044
Gender	.180 (.064)	.005	.075 (.050)	.133
Income Status	.155 (.089)	.082	.003 (.059)	.959

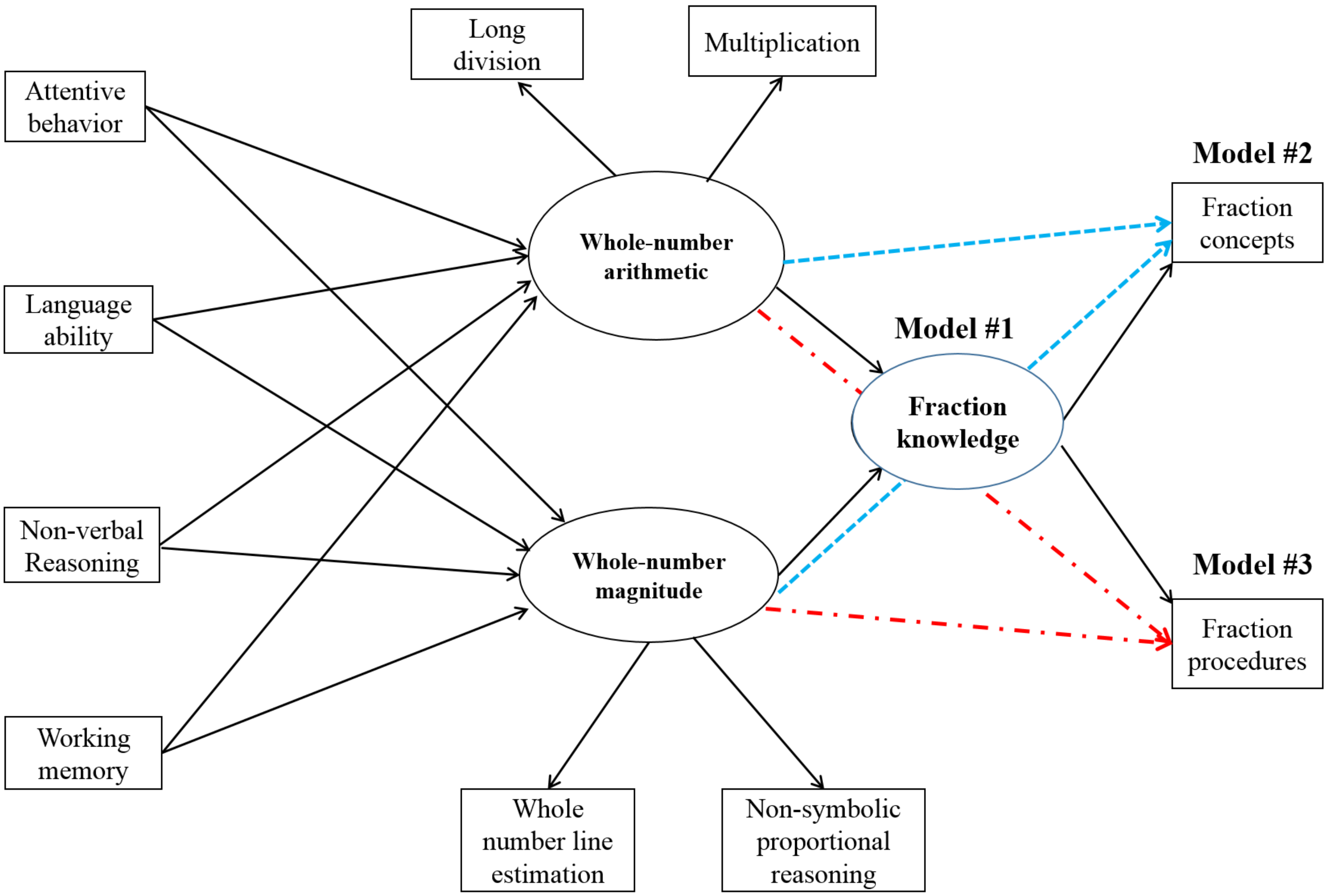
Note: Estimate is standardized coefficients; S.E. = standard error.

Table 7 *Coefficients for Direct Effect (Path a, Path b, and Path c') in Mediation Models*

Model	Path	Coefficient (S.E.)	*p*-value
Predicting fraction knowledge	**Path *a:* Effect of domain general predictors on whole number related mediators**		
	Attentive behavior -> Numerical reasoning	-.270 (.061)	.001
	Language ability -> Numerical reasoning	-.260 (.064)	.001
	Non-symbolic proportional reasoning-> Numerical reasoning	-.263 (.063)	.001
	Working memory-> Numerical reasoning	-.209 (.056)	.001
	Attentive behavior -> Whole number calculations	.458 (.055)	.001
	Language ability -> Whole number calculations	.169 (.060)	.005
	Non-symbolic proportional reasoning -> Whole number calculations	.147 (.060)	.015
	Working memory -> Whole number calculations	.148 (.054)	.006
	Path *b:* Effect of whole number related mediators on fraction knowledge		
	Numerical reasoning -> Fraction knowledge	-1.075 (.255)	.001
	Whole number calculations -> Fraction knowledge	.466 (.091)	.001
	Path *c'*: Direct effect of domain general predict on fraction knowledge		
	Attentive behavior -> Fraction knowledge	-.153 (.113)	.178
	Language ability -> Fraction knowledge	-.056 (.105)	.594
	Non-symbolic proportional reasoning-> Fraction knowledge	-.165 (.105)	.114
	Working memory-> Fraction knowledge	-.132 (.087)	.129
Predicting fraction concepts	**Path *a:* Effect of domain general predictors on whole number related mediators**		
	Attentive behavior -> Numerical reasoning	-.264 (.062)	.001
	Language ability -> Numerical reasoning	-.257 (.064)	.001
	Non-symbolic proportional reasoning-> Numerical reasoning	-.265 (.064)	.001
	Working memory-> Numerical reasoning	-.204 (.057)	.001
	Attentive behavior -> Whole number calculations	.454 (.056)	.001
	Language ability -> Whole number calculations	.167 (.061)	.006
	Non-symbolic proportional reasoning-> Whole number calculations	.145 (.016)	.016
	Working memory-> Whole number calculations	.146 (.007)	.007
	Path *b:* Effect of whole number related mediators on fraction concepts		
	Numerical reasoning -> Fraction concepts	-1.204 (.438)	.006
	Whole number calculations -> Fraction concepts	-.135 (.280)	.631
	Path *c'*: Direct effect of domain general predict on fraction concepts		
	Attentive behavior -> Fraction concepts	.011 (.088)	.897
	Language ability -> Fraction concepts	-.003 (.110)	.975
	Non-symbolic proportional reasoning-> Fraction concepts	-.116 (.116)	.317
	Working memory-> Fraction concepts	-.076 (.088)	.386
Predicting fraction procedures	**Path *a:* Effect of domain general predictors on whole number related mediators**		
	Attentive behavior -> Numerical reasoning	-.262 (.063)	.001
	Language ability -> Numerical reasoning	-.266 (.065)	.001
	Non-symbolic proportional reasoning-> Numerical reasoning	-.255 (.064)	.001
	Working memory-> Numerical reasoning	-.217 (.057)	.001
	Attentive behavior -> Whole number calculations	.459 (.056)	.001
	Language ability -> Whole number calculations	.169 (.061)	.006
	Non-symbolic proportional reasoning-> Whole number calculations	.133 (.061)	.029
	Working memory-> Whole number calculations	.150 (.054)	.006
	Path *b:* Effect of whole number related mediators on fraction procedures		
	Numerical reasoning -> Fraction procedures	-.349 (.247)	.158
	Whole number calculations -> Fraction procedures	.515 (.169)	.002
	Path *c'*: Direct effect of domain general predict on fraction procedures		
	Attentive behavior -> Fraction procedures	.033 (.073)	.649
	Language ability -> Fraction procedures	-.048 (.077)	.537
	Non-symbolic proportional reasoning-> Fraction procedures	-.107 (.075)	.153
	Working memory-> Fraction procedures	-.072 (.062)	.244

Note: Estimate is standardized coefficients; S.E. = standard error. Whole number line estimation is coded as percent absolute error, therefore, higher scores indicate poorer performance.

Figure 1 *Hypothetic Models*



Note: All three models used the same sets of cognitive predictors and latent whole-number meditors (as noted by the solid black arrows); however, Model #1 is predicting the latent variable "fraction knowledge" obtained from fraction concepts measure and fraction procedures measure (as noted by the solid black arrows); Model #2 is predicting fraction concepts (as indicated by blue dotted arrows); Model #3 is predicting fraction procedures (as indicated by red dotted arrows); covariate variable and corresponding paths are not included but were not included in this figure.